Obfuscation Around Point Functions

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Assumptions in Cryptography

Polynomial reduction:
If there exists an algorithm breaking my cryptosystem in polynomial time than it is also possible to solve some well-known hard problem in polynomial time

Cryptographic assumption in this case states that this well-known hard problem has no polynomial solution.

Some classical assumptions:

- Factoring Integers is hard (no polynomial algorithm)
- Discrete Logarithm is hard
- One-Way Functions exist

Outline

Today we discuss obfuscations for several function families, which are black-box secure up to various cryptographic assumptions:

1. Hiding Functionality — Varnovsky, Zakharov, 2003
2. Obfuscating Access Control System — Linn, Prabhakaran, Sahai, 2004
3. Point Functions on a Different Assumption — Wee, 2005
4. Obfuscating "Check-Your-Answer" Procedure — Canetti, 1997

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Property Hiding

Property hiding informally:

- Function family F
- Property $\pi : F \to \{0,1\}$
- Given $O(f)$ it is hard to find $\pi(f)$

More formally:

$$\forall A : |Pr\{A(O(f)) = \pi(f)\} - 1/2| = \nu(|f|)$$

Question: differences with black-box security?

Is There Hidden Functionality in the Program?

prog π_1^w;
var x:string, y:bit;
input(x);
if $x = w$ then y:=1 else
y:=0;
output(y);
end of prog;

prog π_0;
var x:string, y:bit;
input(x);
y:=0; output(y);
end of prog;

Task: Make this families indistinguishable.

Some Theoretical Background

One-Way Permutation is bijection from the set of all binary strings of length k to itself which is easy to compute and difficult to inverse.

$$F : B^k \to B^k$$

Hardcore Predicate for one way permutation F is a predicate (i.e. boolean function) h such that given $F(x)$ its difficult to predict $h(x)$ better than just guess it.

Usual construction of hard-core predicate: choose r by random and take any one way permutation F than given a pair $(F(x), r)$ its difficult to uncover $x \cdot r$.

Obfuscation for Hidden Functionality

prog Π
var x: string, y:bit;
const u, v:string, σ:bit;
input(x);
if ONE_WAY(x)=v then
    if $x \cdot u = \sigma$ then y:=1 else y:=0;
else y:=0;
output(y);
end of prog;

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Point Functions

The family of Point function

$$P_a(x) = 1 \text{ iff } x = a$$

Point functions with output

$$P_{a,b}(x) = b \text{ iff } x = a$$

Multi-point functions with output

$$P_{A,B}(x) = B_i \text{ iff } x = A_i$$

Random Oracle Model

Random function $R : B^n \to B^m$ is just a random element from the set of all functions from B^n to B^m

In Random Oracle Model all participants (obfuscator, obfuscated program and adversary) have oracle access to a random function

Obfuscating Point Functions

- Point functions: store $R(a)$
- Point functions with output: choose random r, store $R_1(a, r)$ and $R_2(a, r) \oplus b$
- Multiple points: repeate above for each point with different r

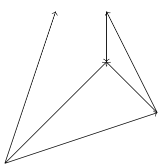
The black-box security is proven for this obfuscation. But some other property of obfuscation is broken.

Which one?

Access Control Mechanism (1)

Informally:

- An unknown graph defining access to nodes
- Each edge has a password
- Start at start node
- Exponentially many access patterns



Start

Access Control Mechanism (2)

- A directed multi-graph G on k vertices. $E = \{(u, v, i) : v = \mu_u^{(i)}\}$
- A set of passwords $\{\pi_e | e \in E\}$
- A set of secrets at the nodes $\{\sigma_v | v \in [k]\}$

$$X_G((i_1, x_1), \dots, (i_n, x_n)) = \begin{cases} v_n, \sigma_{v_n}, & \text{if } \exists v_0, \dots, v_n \in [k] \text{ and} \\ & e_0, \dots, e_{n-1} \in E \text{ such that} \\ & v_0 = 1, e_j = (v_j, v_{j+1}, i_j), \\ & \text{and } x_j = \pi_{e_j} \\ , & \text{otherwise} \end{cases}$$

Claim: obfuscation of access functions can be reduced to that of multi-point functions

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New Assumption

There exists a polynomial-time computable permutation $\pi : B^n \to B^n$ and a constant c such that for every polynomial $s(n)$ and every adversary A of size $s(n)$ for all sufficiently large n,

$$Pr[A(\pi(x)) = x] \leq s(n)^c / 2^n$$

New Construction

Instead of $R(a)$ we will store:

$$h(x, \tau_1, \ldots, \tau_{3n}) = (\tau_1, \ldots, \tau_{3n}, \langle \pi(x), \tau_1 \rangle, \ldots, \langle \pi^{3n}(x), \tau_{3n} \rangle)$$

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Quiz in a Newspaper

- We publish some problem in a newspaper
- We want publish some additional check-yourself information
- We want that this information will contain almost zero information about the answer

Number-theoretic Construction

Let the answer is x
We will publish:

$$H(x) = (r^2, r^{2h(x)})$$

If hash function h is collision-free, then H is black-box secure about x.

Assumption: Let $p = 2q - 1$ and $a, b, c \in_R Z_q^*$. Then distributions $\langle g^a, g^b, g^{ab} \rangle$ and $\langle g^a, g^b, g^c \rangle$ are computationally indistinguishable

Hash-based Construction

Let the answer is x
We will publish:

$$H(x) = (r, h(r, h(x)))$$

Home Problem 2 and 3

2. Prove that probability of changing functionality in obfuscating point functions by using random oracle from B^n to B^{3n} is negligible

3. Prove that $s^c(n)/2^n$, where s is a polynomial and c is a constant, is a negligible function

Summary

Main points:

- It is possible to obfuscate point functions with black-box security
- Security proofs are based on various cryptographic assumptions: existence of one-way functions, Diffie-Hellman indistinguishability, random oracle model
- To be honest, all solutions obsuscate data rather than algorithm

Reading List

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Towards realizing random oracles: Hash functions that hide all partial information, 1997.
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B. Linn, M. Prabhakaran, A. Sahai
Positive results and techniques for obfuscation, 2004.
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On obfuscating point functions, 2005.
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Thanks for attention. Questions?