Decidability of Parameterized Probabilistic Information Flow

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- And somebody observes a **part of its behavior**
- We fix some **property** of the system

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Our result: there is an algorithm answering the above question given any system and property

Outline

1. Information Flow: Definitions
   - System
   - Observation
   - System Properties
   - Definition of Information Flow

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2. Decidability Results

Part I

What is a system?

What is a partial observation of its behavior?

What is a property of the system?

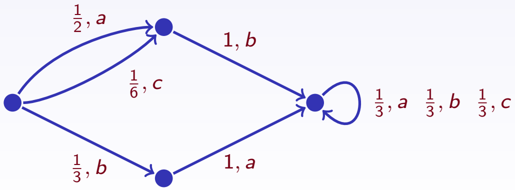
When does a system have **information flow**?

System is a Probability Distribution

- A system is a probability distribution over traces
- A trace is a finite or infinite sequence of alphabet characters
- Today: $\Sigma = L \cup H$ (low-level and high-level events)
- The distribution is described by **Finite Markov Automaton**

Finite Markov Automaton

- Finite number of states
- Edges are labelled by alphabet characters
- Every edge has a probability
- For every edge the sum of probabilities over all outgoing edges is equal to 1



Observation model

For any trace α observation is a projection to low-level events $\alpha|_L$

Projection is just deleting all characters from H from the sequence

Defining a System Property

We describe any property on traces by **recognizing automaton**: property holds $\Leftrightarrow$ automaton accepts

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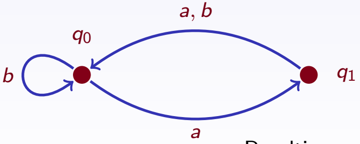
Today we restrict ourselves to properties recognized by Muller automaton

Muller Automaton

- Finite number of states
- Initial state, family $\mathcal{F}$ of "accepting" sets of states
- Every edge is labelled by alphabet character
- The automaton is complete and deterministic: for every pair (v, a) there exist a unique outgoing edge from the vertex v with that label a
- Muller automaton accepts trace if during "reading" it the set of states visited infinitely many times belongs to $\mathcal{F}$



$\mathcal{F} = \{\{q_0, q_1\}\}$

Resulting property: infinite number of a's in the trace

Property-Specific Information Flow

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System S has **no information flow for property** P if

$$\forall u \quad \mathcal{P}r_S(P|u) \; = \; \mathcal{P}r_S(P)$$

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A system is without information flow iff it has no flow for every (defined by Muller automaton) property P

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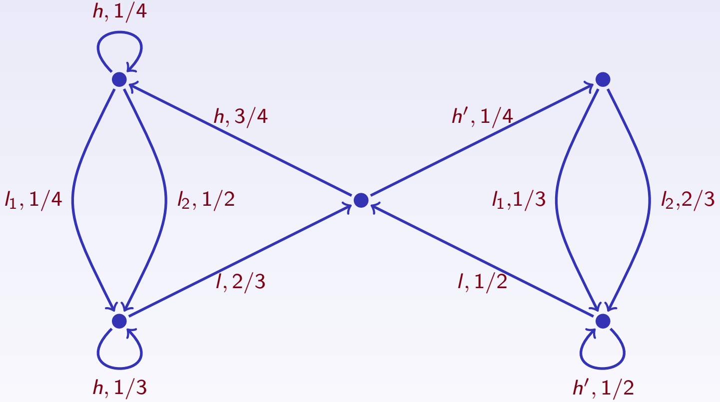
A system is without sequential information flow iff it has no flow for every sequential property P

Information Flow: Example



The Markov chain above has no sequential information flow

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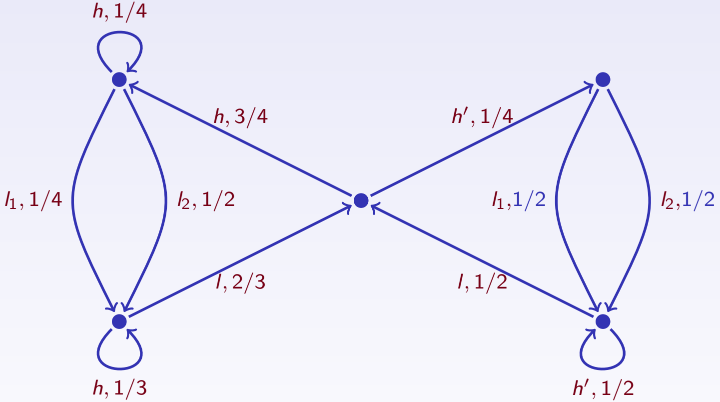

Now the Markov chain above has sequential information flow

Part II

Given system/property can we determine the existence of information flow?

Deciding Property-Specific Information Flow

Theorem

There is an algorithm deciding property-specific information flow for every pair of system/property (i.e. for pair of Muller automaton and Markov chain)

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Reduction to linear algebra:

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Reduction to linear algebra:

1. Compute a composition of Markov automaton and Bučhi automaton
2. Simplify it by the rule "$H^* l \longrightarrow l$"

Algorithm Inside (1/2)

We reduce property-specific information flow to the following mathematical problem:

Input: vectors a, c, matrices $M_1, \ldots, M_n$

Question: does there exist a finite sequence of indices such that $aM_{i_1} \ldots M_{i_k} c \neq 0$?

Algorithm Inside (2/2)

For every k we will compute basis for linear hull of
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3. Stopping condition: $\dim(V_{k+1}) = \dim(V_k)$
4. Check whether $V_k \perp c$

Deciding General Information Flow

Theorem

There is an algorithm deciding general information flow for every system described by a Markov chain

Theorem

There is an algorithm deciding sequential information flow for every system described by a Markov chain

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Future work

- More general models for systems and properties
- Quantitative measure for information flow

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Thanks for your attention! Questions?

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Some related work:

D. Beauquier, M. Duflot, Y. Lifshits
Decidability of Parameterized Probabilistic Information Flow. CSR'07.
http://yury.name/papers/beauquier2007decidability.pdf

D. Beauquier, M. Duflot, M. Minea
A Probabilistic Property-Specific Approach to Information Flow. MMM-ACNS'05.
http://www.univ-paris12.fr/lacl/Rapports/publications/TR-2005-02.pdf

A. Slissenko
Complexity problems in the analysis of information systems security. MMM=ACNS'03.
http://www.springerlink.com/index/WKDENHGBAFE28KNC.pdf