Large Scale Graph Algorithms

A Guide to Web Research: Lecture 2

Yury Lifshits

Steklov Institute of Mathematics at St.Petersburg

Stuttgart, Spring 2007

Talk Objective

To pose an abstract computational problem on graphs that has a huge list of applications in web technologies

Outline

1. Family of Problems: Finding Strongest Connection
   - Problem Statement and Applications
   - Variations of Strongest Connection Problem

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   - Problem Statement and Applications
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   - Statement and Naive Solutions
   - Hierarchical Schema Solution
3. Concluding Remarks
   - Overview of Related Research
   - Open Problems

Part I

Family of Problems: Finding Strongest Connection

Problem statement
Applications
Variations of the problem

Strongest Connection Problem (SCP)

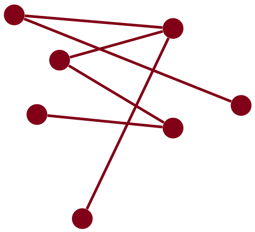
BASIC SETTINGS: a class of graphs $\mathcal{G}$, a class of paths $\mathcal{P}$

INPUT: a graph $G \in \mathcal{G}$
Allowed time for preprocessing: $o(|G|^2)$

QUERY: a (new) vertex v
TASK: to find a vertex $u \in G$
that has maximal number of $\mathcal{P}$-paths from v to u
Allowed time for query processing: $o(|G|)$

Homogeneous Graph / 2-Step Paths

Graph of coauthoring



Coauthor suggest in **DBLP**

The most common coauthor of my coauthors

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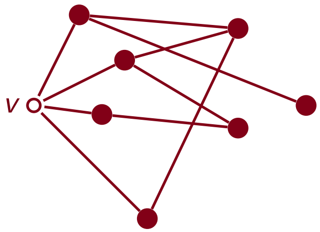

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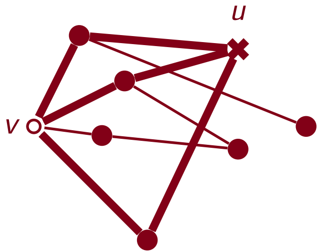

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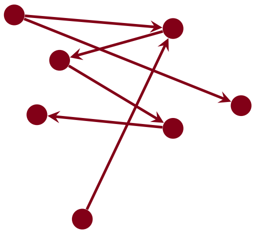
Graph of hyperlinks



Advanced option for **Google search**: link-based similar website

The website that is most often co-cited with the given one

Directed Graph / 2-Step Paths

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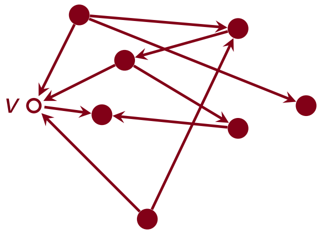

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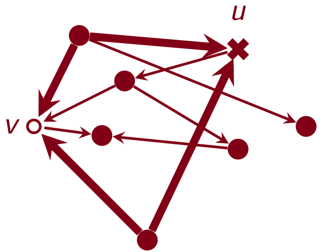

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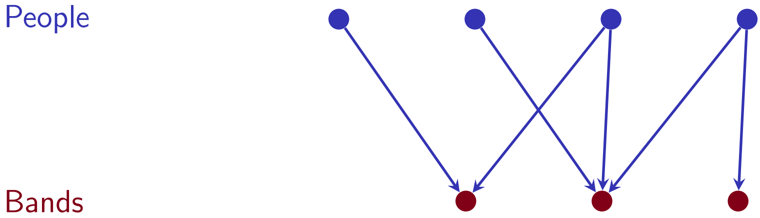
Bipartite Graph / 2-Step Paths

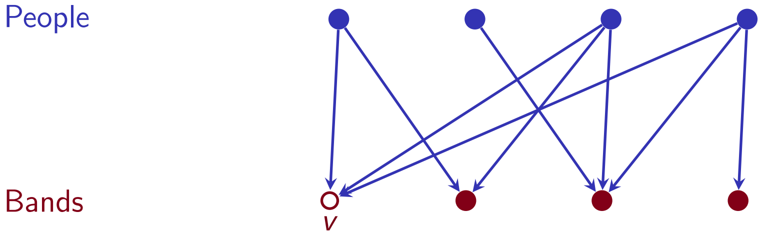


Last.fm similar music bands

The band that is most often co-listened with the given one

In general: any content-based similarity, keyword-similarity, any co-occurrence similarity

Bipartite Graph / 2-Step Paths



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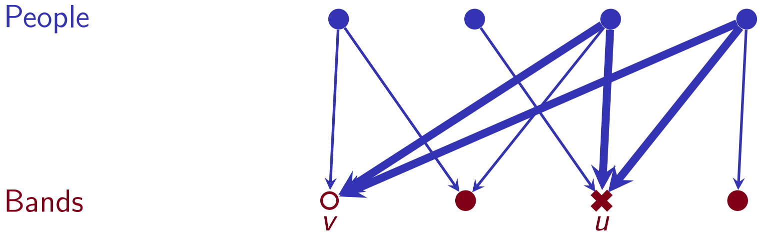

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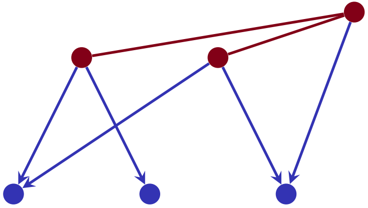
Homogeneous-Bipartite Graph / 2-Step Paths

Friendship graph

Recommended items



Social recommendations in networks like **Facebook**

System recommends things that are popular among my friends

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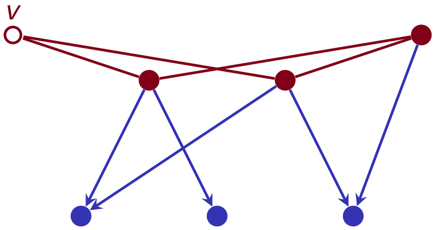

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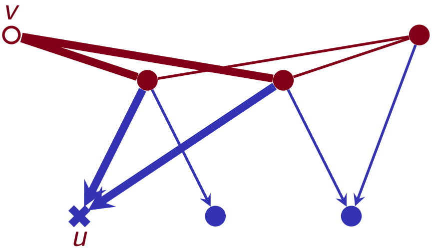

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Bipartite Graph / 3-Step Paths

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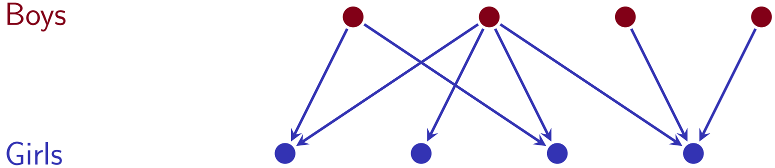

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Subscription recommendations for **FeedBurner**, **Google Reader**

Items that have the largest number of co-occurrences with my items

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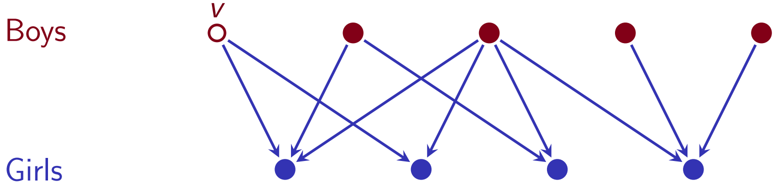

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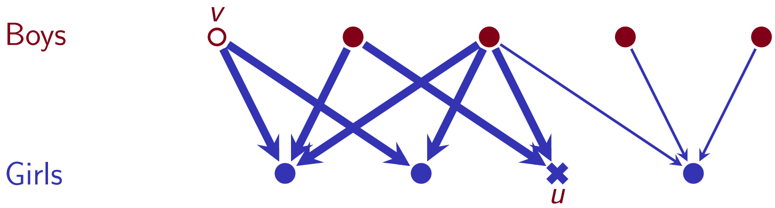

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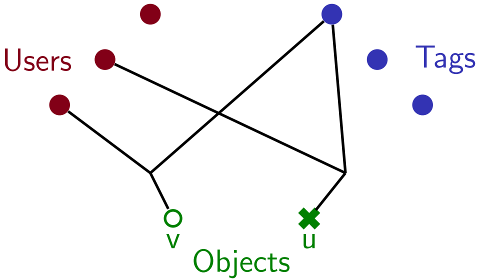
Tripartite 3-Graph / 2-Step Paths

Folksonomy is a set of triples $< user, tag, object >$



Similar websites in **Del.icio.us**, similar pictures in **Flicr**

Largest number of common tags
Largest number of common users
Largest number of common pairs $< user, tag >$

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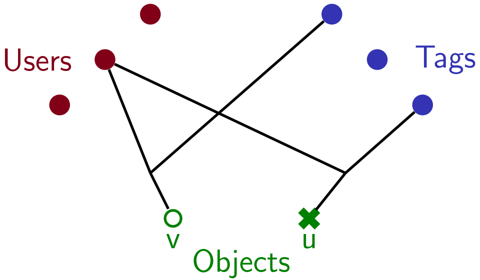

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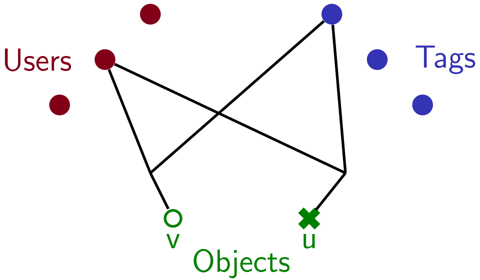

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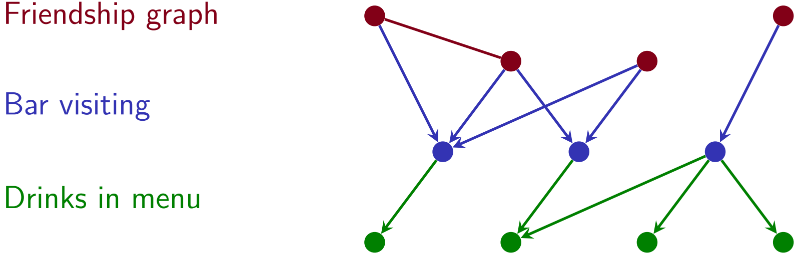
Multicolor-Multiparty Graph / k-Step Paths

Semantic search: "Most popular drink that is available on bars that are visited by my friends"



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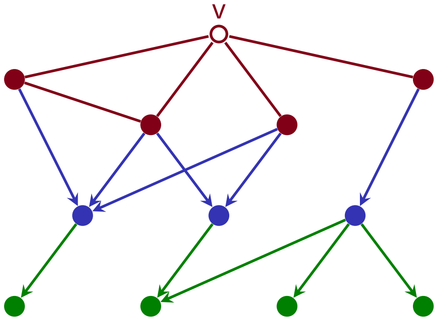
Bar visiting

Drinks in menu



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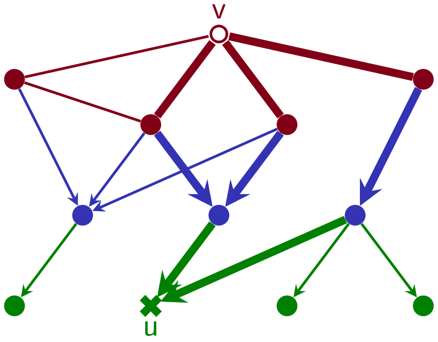

Variations of Strongest Connection Problem

- Directed/undirected graphs
- Weights on edges/vertices
- Task: offline, on-line, all-to-all
- Task: one best connection, k best connections
- Graph and weights are evolving with time

Claim

Computing strongest connection is probably the most important algorithmic problem related to web technologies

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Computing strongest connection is probably the most important algorithmic problem related to web technologies★

★Personal opinion of Yury Lifshits

Solution Variations

Usual alternatives to exact algorithm:

- Approximate algorithms
- Randomized algorithms
- Input graph (or query) belongs to a certain distribution. Average complexity analysis
- Introducing additional assumptions
- Introducing additional input-complexity parameter
- Modifying the computation task
- Heuristics
- Look to particular cases (subproblems)

Part II
Max-Intersection Problem

Statement and naive solutions
Hierarchical schema solution

This section represents a work-in-progress joint research with Benjamin Hoffmann and Dirk Nowotka

Statement of Max-Intersection Problem

In set notation:

Input: Family $\mathcal{F}$ of n sets, $\forall f \in \mathcal{F} \quad |f| \leq k$
Time for preprocessing: $n \cdot polylog(n) \cdot poly(k)$

Query: a set f_{new}, $|f_{new}| \leq k$
Task: Find $f_i \in \mathcal{F}$ that maximizes $|f_{new} \cap f_i|$
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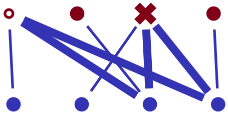

In bipartite graph notation:

Input: Bipartite documents-terms graph, $|\mathcal{D}| = n$, $\forall d \in \mathcal{D} \quad |d| \leq k$

Query: a document d_{new}, $|d_{new}| \leq k$
Task: Find $d_i \in \mathcal{D}$ that has maximal number of common terms with d_{new}

Applications of Max-Intersection (1/2)

Homogeneous graphs:

- References in scientific papers: (1) maximal number of co-occurrences in reference list (2) maximal intersection of reference lists
- Social networks (e.g. LinkedIn): a person that has maximal connections with my direct neighborhood
- Collaboration networks (e.g. DBLP): given a scientist, to find another one with maximal overlapping of coauthors-list

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Bipartite graphs:

- Websites—Words graph: find a website with maximal intersection of used terms with the given one
- Music_Bands—Listeners graph: find a band that has maximal intersection of listeners with the given one

Applications of Max-Intersection (2/2)

Tripartite graphs:

- Long_Search_Queries—Web_Dictionary—Websites: given a query to find a website with maximal number of query terms
- Advertisement_Description—Keywords—Websites (e.g. AdSense Matching): find a website with maximal number of terms form advertisement description
- PC_Members—Keywords—Submissions: find a paper that has maximal number of terms that belong to expertise of the given PC member

Inverted Index (1/2)

Let us use documents-terms notation

Inverted index approach:

- Preprocessing. For every term produce a list of all documents that contain it

  Complexity: $O(n \cdot k)$

- Query $d_{new} = \{t_1, \ldots, t_k\}$. Retrieve document lists for all terms of query. Check all documents in all these k lists and return the one with maximal intersection with d_{new}

  Worst case complexity: $\Omega(n)$

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Let T_{max} be the maximal degree of terms. Then the query complexity is $O(k \cdot T_{max})$

Inverted Index (2/2) Rare-Term Requirement

Cheating: modify the Max-Intersection problem

New Task: Given the document d_{new}, find a document d_i such that

1. It has a joint **rare term** (term that occurs in at most r documents) with d_{new}
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Observation

Inverted index can handle queries in $O(r \cdot k)$ time now

Inverted Set-Index

Assume that k is extremely small, say $k = O(\log\log n)$

Inverted set-index approach:

- Preprocessing. Write down all term subsets of all documents. Sort all these subsets in lexicographical order

  Complexity: $O(n \cdot 2^k)$

- Query $d_{new} = \{t_1, \ldots, t_k\}$. For every subset of query terms search it in the inverted set-index. Return the document that corresponds to the maximal subset founded in index

  Complexity: $O(2^k(k + \log n))$

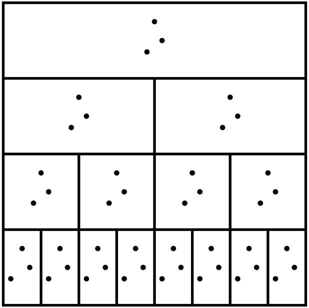
Hierarchical Schema

Table of terms:

k levels
Level i is divided to 2^{i-1} cells
Every cell contains k terms

Random nature of $\mathcal{D}$ and d_{new}:

Choose random cell on the bottom level
Mark all cells that are above it
Choose one random term in every marked cell



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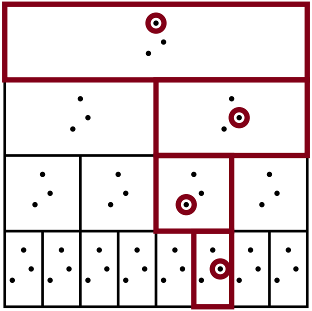

Magic Levels (1/2)

Assume that there are 2^k such "random" documents in $\mathcal{D}$

Notation: **magic levels** $q = \frac{k}{\log k+1}, \quad q' = \frac{k}{\log k}$

Theorem

With very high probability there exists $\mathbf{d} \in \mathcal{D}$ that has the same terms from top $\mathbf{q} - \varepsilon$ levels

Magic Levels (2/2)

Theorem

With very high probability there are no $d \in \mathcal{D}$ that has at least $q' + \varepsilon$ common elements with d_{new}

Algorithm for Hierarchical Schema

Preprocessing:

Encode every document as a $2k - 1$ sequence,
every odd element lies in range $[1..k]$, every even is 0 or 1
Construct a lexicographic tree for all encodings

Query processing:

Find the largest **prefix-match** between d_{new} and documents from $\mathcal{D}$

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By two theorems above with very high probability maximal prefix-match is very close to maximal intersection

Part III
Concluding Remarks

Overview of related research

Open problems

Overview of Related Research

Famous computational problems that need scalable algorithms:

- Nearest neighbors in vector spaces
- Nearest neighbors in abstract metric spaces
- Connection subgraph problem
- Collaborative filtering
- Mining association rules
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Alternative: randomized model of input + probabilistic analysis

Alternative: realistic assumption about input + exact algorithm

Algorithms for Max-Intersection

Algorithmic open problems:

1. Max-Intersection for bounded tree-width graphs
2. Max-Intersection in configuration model
3. Max-Intersection in preferential attachment model

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Long-term goal: to develop **theoretical** framework for scalability analysis of algorithms

Data Structure Complexity

On-line inclusion problem

Input: Family $\mathcal{F}$ of 2^k subsets of $[1..k^2]$

Data storage after preprocessing: $2^k \cdot poly(k)$

Query: a set $f_{new} \subseteq [1..k^2]$
Task: decide whether $\exists f \in \mathcal{F} : \quad f_{new} \subseteq f$

Time for query processing: $poly(k)$

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Conjecture: the on-line inclusion problem **can not** be solved within such time/space constraints

Call for participation

Know a relevant reference?

Have an idea?

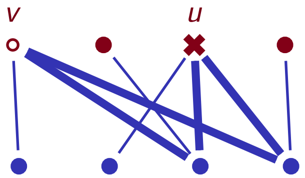
Find a mistake?

Solved one of these problems?

- Knock to my office 1.156
- Write to me `yura@logic.pdmi.ras.ru`
- Join our informal discussions
- Participate in writing a follow-up paper

Highlights

Strongest Connection family, including **Max-Intersection**



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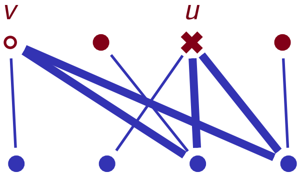

Open problems:

Max-Intersection in **complex-networks models**

Data structure complexity of **on-line inclusion problem**

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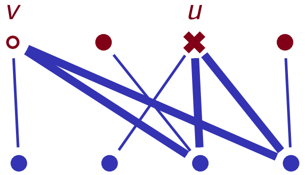

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Vielen Dank für Ihre Aufmerksamkeit! Fragen?

References (1/2)

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